FINAL REPORT PROGRAM LEFE

Program LEFE / MANU	**Project Title** **D-WAVE**	Years 2019 – 2021

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ANR NABUCO

Context

Representation and description of hydrodynamic mechanisms in oceanography. Since the direct resolution of the Navier-Stokes equations is still out of reach from an operational point of view, we are interested in developing depth averaged models (Shallow-Water or dispersive type), accurate and computationally efficient.

Objectives / scientific questions

Improvement of the existing models at the physical level (description of wave-breaking, sediment transport), numerical (appropriate discrete formulations, hyperbolisation) and implementation (parallelization, implementation on hybrid architectures).

Main results

Breaking in the context of wave trains.

This point was one of the main objectives of the project, which aimed to extend the method of Kazakova & Richard, 2019 to wave trains on general topographies. This work has been subject of the publication [P1], and of two conference proceedings [C1,C2].

Taking into account the Coriolis force in the Shallow-Water equations.

This work was carried out during the CEMRACS summer school as part of a project co-financed by this LEFE program. The project aimed to extend a method to ensure geostrophic equilibrium in the framework of linear Shallow-Water equations to the non-linear case. Several proposals on collocated grids and staggered meshes have been proposed in the paper [P2].

Implementation of the numerical model of Marche, 2019 on hybrid architectures.

This axis is one of the major challenges of the project. This work is still in progress within the framework of a collaboration between the ICJ (A. Duran, B. Fabrèges) and the IMAG (F. Marche). To date, the hyperbolic part of the problem has been exported to hybrid architecture. The integration of the dispersive part and the wave-breaking model remain to be achieved. Work is underway in this direction, and should be supported from September 2022 by a DGA funding obtained by F. Marche and A. Duran.

Hyperbolic dispersive models.

In parallel with this work, G.L. Richard proposed in a recent paper a hyperbolic compressible model allowing, in a certain asymptotics, to recover the classical fully nonlinear dispersive model of Green-Naghdi [3]. The prospects in terms of modeling and computational time are considerable. Two papers are being written, one as part of Khawla M'Sheik's post-doctorate on the inclusion of elastic effects and a bi-layer extension, and the other on the numerical implementation of the model, with a view to comparisons with the classic Discontinuous Galerkin (DG) version developed by F. Marche in 2019.

Stability and convergence analysis of dG methods.

D. LeRoux continued his work on the stability and convergence of approximate solutions obtained by discretizing equations from geophysics with discontinuous Galerkin (DG) type numerical methods. For the Korteweg - de Vries (KdV) equation he showed in [P4] which stability parameters are necessary and sufficient to guarantee the stability of the method and obtained a superconvergence result for the discrete frequency. In [P5] he proposed a DG method based on nonconforming elements. He obtained a superconvergence result for the discrete frequency, in fact two orders more than for a P1DG method and for half the computational cost.

Future of the project :

- **Numerical implementation of the dG model developed by Marche, 2019.** Continuation of parallelization and export work on hybrid CPU/GPU architectures. Implementation of the dG discrete formulations developed by D. LeRoux during the project. This work now benefits from DGA (French Ministry of Defense) funding as part of a collaboration with SHOM (France's Naval Hydrographic and Oceanographic Service).

- **Wave breaking and sediment transport.** This axis now benefits from funding from the IMPT (Institut des Mathématiques pour la Planète Terre), which notably made it possible to finance Yen Chung Hung's doctorate at LAMA (Univ. of Chambéry), under the supervision of M. Kazakova, an active collaborator on these aspects during the project, and J. Chauchat (LEGI, Grenoble). The thesis focuses on sediment transport and the inclusion of wave breaking in the model recently proposed by G. Richard [P3].

- Hyperbolic dispersive models. The issues related to numerical implementation are being studied as part of a collaboration with SHOM and INSA Toulouse. A paper is being written, in which we exploit the hyperbolic character of the model [P3] to propose a numerical scheme that is energetically stable, computationally efficient and capable of effectively managing the boundary conditions. A bi-layer extension of the model is being derived as part of the post-doctorate of Khawla M'Sheik at ICJ (Lyon), under the supervision of A. Duran and G. Richard.

Publications

[P1] A. Duran, G.L. Richard - *Modelling coastal wave trains and wave breaking*. Ocean Modelling, Vol. 147, 2020.
[P2] E. Audusse, V. Dubos, A. Duran, N. Gaveau, Y. Nasseri and Y. Penel - *Numerical approximation of the shallow water equations with Coriolis source term,* ESAIM: PROCEEDINGS AND SURVEYS, Vol. 70, p. 31-44, 2021.
[P3] G.L. Richard - *An extension of the Boussinesq-type models to weakly compressible flows*, European Journal of Mechanics - B/Fluids, vol. 89, pages 217-240, 2021.
[P4] Daniel Y. Le Roux - *Fourier analysis of the local discontinuous Galerkin method for the linearized KdV equation*, submitted February 22, 2022.
[P5] Daniel Y. Le Roux - *A super-convergent quasi-discontinuous Galerkin method for transport-dominated problems in geophysical flow*, to be submitted on March 2022.

Proceedings

[C1] A. Duran, B. Fabrèges, M. Kazakova, G.L. Richard - *Modeling Shoaling and breaking waves - Wave trains and two-dimensional applications*. XVIèmes Journées Nationales Génie Côtier – Génie Civil, Le Havre, 2020.
[C2] G. Richard, M. Kazakova, B. Fabrèges, A. Duran - *Coastal waves in a depth-averaged model with an anisotropic turbulence and a weakly compressible approach.* ICTAM conference, 2020.

Communications

Due to sanitary conditions, we were only able to propose to present these works at the two conferences above.

Supervision

Since April 2021 : post-doctorate of K. M'Sheik at ICJ (Lyon), co-supervised by G.L. Richard and A. Duran, with the financial support of ANR NABUCO.